result in equal autarky-relative commodity prices in the two nations. This is assigned as end-of-chapter Problem 4, with the answer at the end of the book.

Note also that the H–O theory does not require identical tastes (i.e., equal indifference curves) in the two nations. It only requires that if tastes differ, they do not differ sufficiently to neutralize the tendency of different factor endowments and production possibility curves from leading to different relative commodity prices and comparative advantage in the two nations. Thus, in a sense, Figure 3.4 can be regarded as a more general illustration of the H–O model than Figure 5.4. Case Study 5-3 examines the pattern of revealed comparative advantage and disadvantage of various countries or regions.

5.5 Factor–Price Equalization and Income Distribution

In this section, we examine the *factor–price equalization theorem,* which is really a corollary, since it follows directly from the H–O theorem and holds only if the H–O theorem holds. It was *Paul Samuelson* (1970 Nobel prize in economics) who rigorously proved this factor–price equalization theorem (corollary). For this reason, it is sometimes referred to as the Heckscher–Ohlin–Samuelson theorem (H–O–S theorem, for short).

In Section 5.5A, we state the theorem and explain its meaning. Section 5.5B presents an intuitive proof of the factor–price equalization theorem. In Section 5.5C, we examine the related question of the effect of international trade on the distribution of income within each trading nation. Finally, in Section 5.5D, we briefly consider the empirical relevance of this theorem. The rigorous proof of the factor–price equalization theorem is presented in the appendix to this chapter. The proof requires the tools of analysis of intermediate microeconomic theory presented in the appendix to Chapter 3.

5.5A The Factor–Price Equalization Theorem

Starting with the assumptions given in Section 5.2A, we can state the **factor–price equalization (H–O–S) theorem** as follows: *International trade will bring about equalization in the relative and absolute returns to homogeneous factors across nations.* As such, international trade is a substitute for the international mobility of factors.

What this means is that international trade will cause the wages of homogeneous labor (i.e., labor with the same level of training, skills, and productivity) to be the same in all trading nations (if all of the assumptions of Section 5.2A hold). Similarly, international trade will cause the return to homogeneous capital (i.e., capital of the same productivity and risk) to be the same in all trading nations. That is, international trade will make w the same in Nation 1 and Nation 2; similarly, it will cause r to be the same in both nations. Both relative and absolute factor prices will be equalized.

From Section 5.4, we know that in the absence of trade the relative price of commodity X is lower in Nation 1 than in Nation 2 because the relative price of

labor, or the wage rate, is lower in Nation 1. As Nation 1 specializes in the production of commodity X (the *L*-intensive commodity) and reduces its production of commodity Y (the *K*-intensive commodity), the relative demand for labor rises, causing wages (w) to rise, while the relative demand for capital falls, causing the interest rate (r) to fall. The exact opposite occurs in Nation 2. That is, as Nation 2 specializes in the production of Y and reduces its production of X with trade, its demand for L falls, causing w to fall, while its demand for K rises, causing r to rise.

To summarize, international trade causes w to rise in Nation 1 (the low-wage nation) and to fall in Nation 2 (the high-wage nation). Thus, international trade reduces the pretrade difference in w between the two nations. Similarly, international trade causes r to fall in Nation 1 (the *K*-expensive nation) and to rise in Nation 2 (the *K*-cheap nation), thus reducing the pretrade difference in r between the two nations. This proves that international trade *tends to reduce* the pretrade difference in w and r between the two nations.

We can go further and demonstrate that international trade not only tends to reduce the international difference in the returns to homogeneous factors, but would in fact bring about complete equalization in relative factor prices when all of the assumptions made hold. This is so because as long as relative factor prices differ, relative commodity prices differ and trade continues to expand. But the expansion of trade reduces the difference in factor prices between nations. Thus, international trade keeps expanding until relative commodity prices are completely equalized, which means that relative factor prices have also become equal in the two nations.

5.5B Relative and Absolute Factor–Price Equalization

We can show graphically that relative factor prices are equalized by trade in the two nations (if all the assumptions of Section 5.2A hold). In Figure 5.5, the relative price of labor (w/r) is measured along the horizontal axis, and the relative price of commodity X (P_X/P_Y) is measured along the vertical axis. Since each nation operates under perfect competition and uses the same technology, there is a one-to-one relationship between w/r and P_X/P_Y. That is, each w/r ratio is associated with a specific P_X/P_Y ratio.

Before trade, Nation 1 is at point A, with $w/r = (w/r)_1$ and $P_X/P_Y = P_A$, while Nation 2 is at point A', with $w/r = (w/r)_2$ and $P_X/P_Y = P_{A'}$. With w/r lower in Nation 1 than in Nation 2 in the absence of trade, P_A is lower than $P_{A'}$ so that Nation 1 has a comparative advantage in commodity X.

As Nation 1 (the relatively *L*-abundant nation) specializes in the production of commodity X (*L*-intensive commodity) and reduces the production of commodity Y, the demand for labor increases relative to the demand for capital and w/r rises in Nation 1. This causes P_X/P_Y to rise in Nation 1. On the other hand, as Nation 2 (the *K*-abundant nation) specializes in the production of commodity Y (the *K*-intensive commodity), its relative demand for capital increases and r/w rises (i.e., w/r falls). This causes P_Y/P_X to rise (i.e., P_X/P_Y to fall) in Nation 2. The process will continue until point $B = B'$, at which $P_B = P_{B'}$ and $w/r = (w/r)^\star$ in both

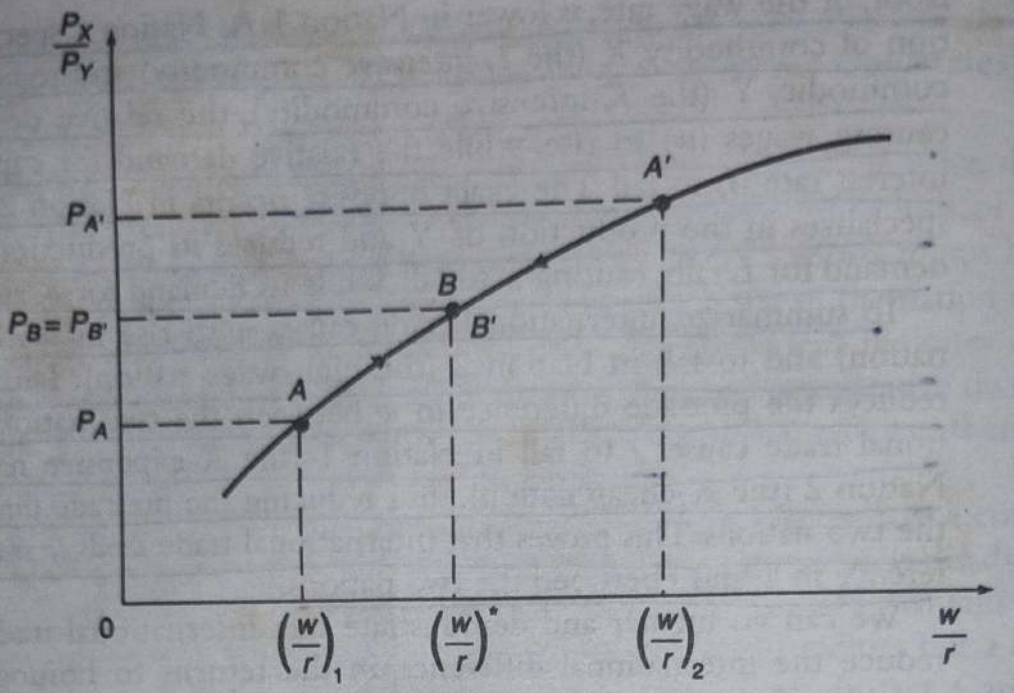

FIGURE 5.5. Relative Factor–Price Equalization. The horizontal axis measures w/r and the vertical axis P_X/P_Y. Before trade, Nation 1 is at point A, with $w/r = (w/r)_1$ and $P_X/P_Y = P_A$ while Nation 2 is at point A', with $w/r = (w/r)_2$ and $P_X/P_Y = P_{A'}$. Since w/r is lower in Nation 1 than in Nation 2, P_A is lower than $P_{A'}$ so that Nation 1 has a comparative advantage in commodity X. As Nation 1 specializes in the production of commodity X with trade and increases the demand for labor relative to capital, w/r rises. As Nation 2 specializes in the production of commodity Y and increases its relative demand for capital, r/w rises (i.e., w/r falls). This will continue until point $B = B'$, at which $P_B = P_{B'}$ and $w/r = (w/r)^*$ in both nations.

nations (see Figure 5.5). Note that $P_B = P_{B'}$ *only if* w/r is identical in the two nations, since both nations operate under perfect competition and use the same technology (by assumption). Note also that $P_B = P_{B'}$ lies between P_A and $P_{A'}$, and $(w/r)^*$ lies between $(w/r)_1$ and $(w/r)_2$. To summarize, P_X/P_Y will become equal as a result of trade, and this will occur only when w/r has also become equal in the two nations (as long as both nations continue to produce both commodities). A more rigorous and difficult proof of the relative factor–price equalization theorem is given in the appendix.

The preceding paragraph shows the process by which *relative, not absolute,* factor prices are equalized. Equalization of *absolute* factor prices means that free international trade also equalizes the real wages for the same type of labor in the two nations and the real rate of interest for the same type of capital in the two nations. However, given that trade equalizes relative factor prices, that perfect competition exists in all commodity and factor markets, and that both nations use the same technology and face constant returns to scale in the production of both commodities, it follows that trade also equalizes the absolute returns to homogeneous factors. A rigorous and difficult proof of absolute factor–price equalization is presented in the appendix to this chapter, following the proof of relative factor–price equalization.

Note that trade acts as a substitute for the international mobility of factors of production in its effect on factor prices. With perfect mobility (i.e., with complete information and no legal restrictions or transportation costs), labor would migrate from the low-wage nation to the high-wage nation until wages in the two nations became equal. Similarly, capital would move from the low-interest to the high-interest nation until the rate of interest was equalized in the two nations. *While trade operates on the demand for factors, factor mobility operates on the supply of factors.* In either case, the result is complete equalization in the absolute returns of homogeneous factors. With some (rather than perfect) international mobility of factors, a smaller volume of trade would be required to bring about equality in factor returns between the two nations.

Sections A5.5 to A5.7 then examine factor-intensity reversal, utilizing the more advanced analytical tools reviewed in the appendix to Chapter 3. Section A5.5 gives a diagrammatic presentation of factor-intensity reversal. Section A5.6 presents the formula to measure the elasticity of substitution of L for K in production and examines its relationship to factor-intensity reversal. Section A5.7 discusses the method used to conduct empirical tests to determine the prevalence of factor-intensity reversal in the real world.

5.1 The Edgeworth Box Diagram for Nation 1 and Nation 2

Figure 5.7 shows the Edgeworth box diagram of Nation 2 superimposed on the box diagram of Nation 1 in such a way that their origins for commodity X coincide. The origins for commodity Y differ because Nation 1 has a relative abundance of labor, whereas Nation 2 has a relative abundance of capital. The box diagrams are superimposed on each other to facilitate the analysis to follow.

Because both nations use the same *technology*, the isoquants for commodity X in the two nations are identical (and are measured from the common origin O_X). Similarly, the isoquants for commodity Y in the two nations are also identical (but are measured from origin O_Y for Nation 1 and from origin $O_{Y'}$ for Nation 2). X-isoquants farther from O_X refer to progressively higher outputs of X, while Y-isoquants farther from O_Y or $O_{Y'}$ refer to greater outputs of Y.

By joining all points where an X-isoquant is tangent to a Y-isoquant in each nation, we obtain the nation's production contract curve. Points A, F, and B on Nation 1's production contract curve in Figure 5.7 refer to corresponding points on Nation 1's production frontier (see Figure 3.9). Similarly, points A', F', and B' on Nation 2's production contract curve refer to corresponding points on Nation 2's production frontier. Note that the contract curves of both nations bulge toward the lower right-hand corner because commodity X is the L-intensive commodity in both nations.

5.2 Relative Factor–Price Equalization

Figure 5.8 repeats Figure 5.7 but omits (to keep the figure simple) all isoquants as well as points F and F' (which are not needed in the subsequent analysis). The no-trade equilibrium point is A in Nation 1 and A' in Nation 2 (as in Figures 3.3 and 3.4). The K/L ratio in the production of commodity X is smaller in Nation 1 than in Nation 2. This is given by the lesser slope of the line (not shown) from origin O_X to point A as opposed to point A'. Similarly, the K/L ratio in the production of commodity Y is also smaller in Nation 1 than in Nation 2. This is given by the smaller slope of the line (not shown) from O_Y to point A as opposed to the slope of the line (also not shown) from $O_{Y'}$ to point A'.

Since Nation 1 uses a smaller amount of capital per unit of labor (K/L) in the production of both commodities with respect to Nation 2, the productivity of labor and therefore the wage rate (w) are lower, while the productivity of capital and therefore the rate of interest (r) are higher, in Nation 1 than in Nation 2. This is always the case when both nations use a production function that is homogeneous of degree one, showing constant returns to scale (as assumed throughout).

With a lower w and a higher r, w/r is lower in Nation 1 than in Nation 2. This is consistent with the relative physical abundance of labor in Nation 1 and capital in Nation 2. The lower w/r in Nation 1 at autarky point A is reflected in the smaller (absolute) slope of the (short and solid) straight line through point A as opposed to the corresponding line at point

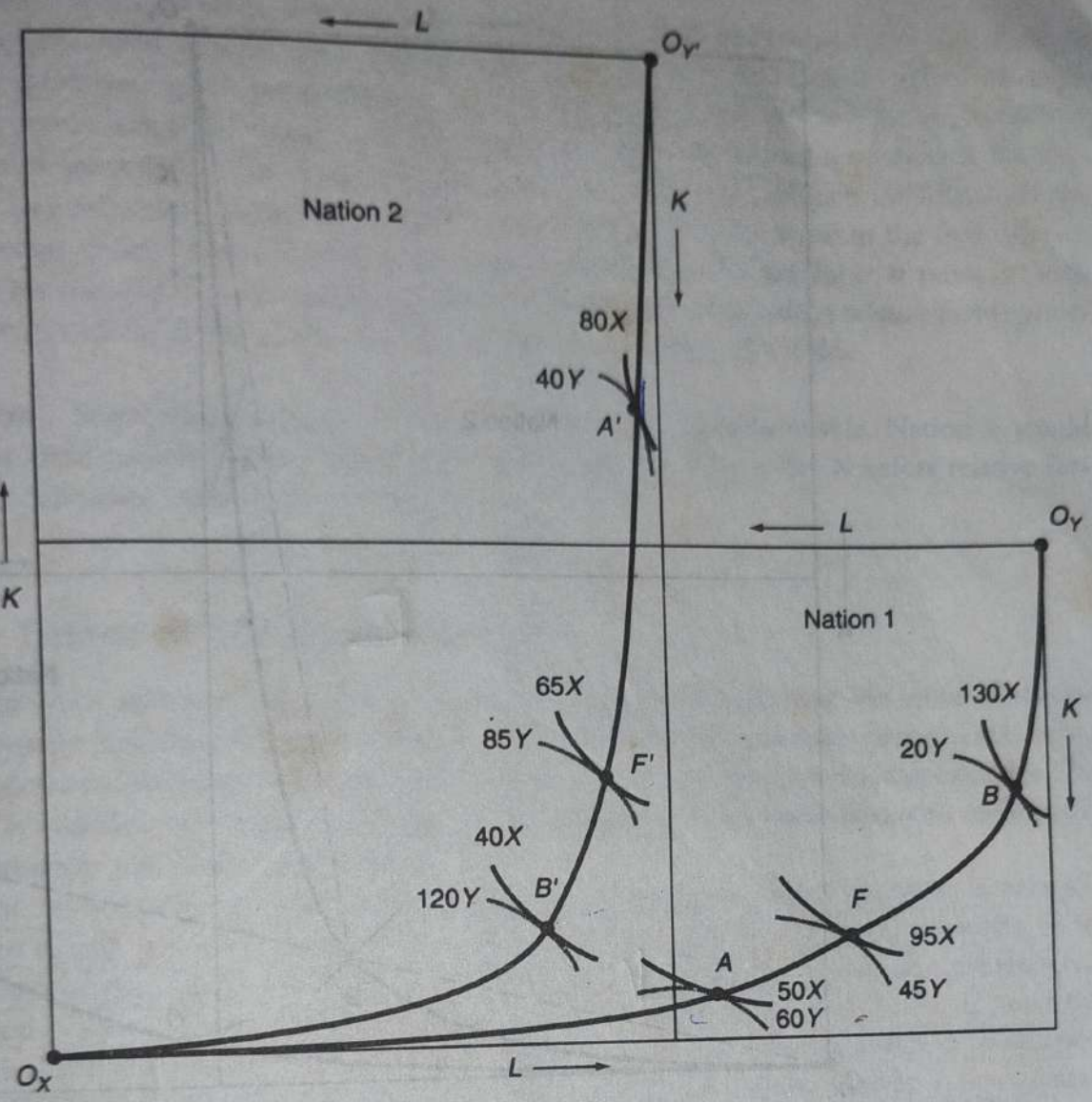

FIGURE 5.7. The Edgeworth Box Diagram for Nation 1 and Nation 2—Once Again. The Edgeworth box diagram of Nation 2 from Figure 3.10 is superimposed on the box diagram for Nation 1 from Figure 3.9 in such a way that their origins for commodity X coincide. Because both nations use the same technology, the isoquants of commodity X are identical in the two nations. The same is true for the Y-isoquants. The points on each nation's production contract curve refer to corresponding points on the nation's production frontier. The contract curves of both nations bulge toward the lower right-hand corner because commodity X is the *L*-intensive commodity in both nations.

A'. (The straight lines are the common tangents to the X- and Y-isoquants—not shown in Figure 5.8—at point *A* and point *A'*.)

To summarize, we can say that at the no-trade equilibrium point *A*, Nation 1 uses a smaller *K/L* ratio in the production of both commodities with respect to Nation 2. This results in lower productivity of labor and higher productivity of capital in Nation 1 than in Nation 2. As a result, *w/r* is lower in Nation 1 (the *L*-abundant nation) than in Nation 2.

Since Nation 1 is the *L*-abundant nation and commodity X is the *L*-intensive commodity, with the opening of trade Nation 1 will specialize in the production of commodity X (i.e., will move from point *A* toward O_Y along its production contract curve). Similarly, Nation 2 will specialize in the production of commodity Y and move from point *A'* toward O_X. Specialization in production continues until Nation 1 reaches point *B* and Nation 2 reaches point *B'*, where *K/L* is the same in each commodity in both nations. This is given

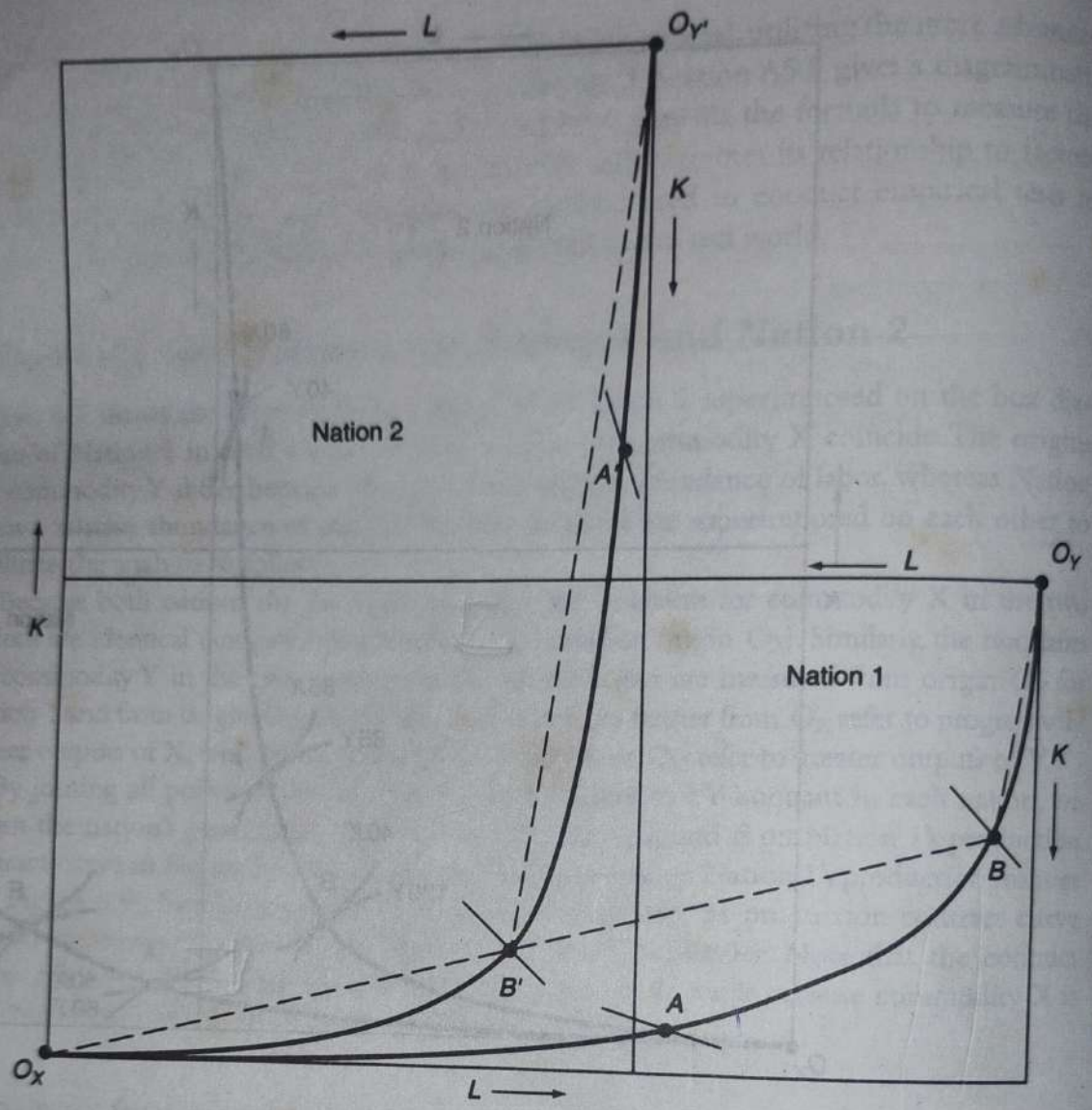

FIGURE 5.8. Formal Proof of the Factor–Price Equalization Theorem. At the no-trade equilibrium point A in Nation 1 and A' in Nation 2, K/L is lower in the production of both commodities in Nation 1 than in Nation 2. These are given by the lower slopes of straight lines (not shown) from O_X and O_Y or $O_{Y'}$ to points A and A'. Since w/r (the absolute slope of the solid line through point A) is lower in Nation 1 and commodity X is L intensive, Nation 1 specializes in the production of commodity X until it reaches point B. Nation 2 specializes in Y until it reaches point B'. At B and B', K/L and therefore w/r are the same in both nations.

by the slope of the dashed line from O_X through points B' and B for commodity X, and by the parallel dashed lines from O_Y and $O_{Y'}$ to points B and B' for commodity Y, for Nation 1 and Nation 2, respectively.

Note that as Nation 1 moves from point A to point B, K/L rises in the production of both commodities. This is reflected by the steeper slope of the dashed lines from O_X and O_Y to point B as opposed to point A. As a result of this increase in K/L, the productivity and therefore the wage of labor rise in Nation 1 (the low-wage nation). On the other hand, as Nation 2 moves from point A' to B', K/L falls in the production of both commodities. This is reflected by the smaller slope of the dashed lines from $O_{Y'}$ and O_X to point B' as opposed to point A'. As a result of this decline in K/L, the productivity and therefore the wage of labor falls in Nation 2 (the high-wage nation). The exact opposite is true for capital.

In the absence of trade, w/r was lower in Nation 1 than in Nation 2 (see the absolute slopes of the solid straight lines through points A and A'). As Nation 1 (the low-wage nation) specializes in the production of commodity X, K/L and w/r rise in the production of both commodities in Nation 1. As Nation 2 (the high-wage nation) specializes in the production of commodity Y, K/L and w/r fall in the production of both commodities. Specialization in production continues until K/L and w/r have become equal in the two nations. This occurs when Nation 1 produces at point B and Nation 2 produces at point B' with trade. This concludes our formal proof that international trade equalizes relative factor prices in the two nations when all the assumptions listed in Section 5.2A hold.

Problem Show graphically that with sufficiently less capital available, Nation 1 would have become completely specialized in the production of commodity X before relative factor prices became equal in the two nations.

A5.3 Absolute Factor–Price Equalization

This proof of absolute factor–price equalization is more difficult than the proof of relative factor–price equalization and is seldom if ever covered in undergraduate courses, even when all students in the course have had intermediate microeconomics and macroeconomics. The proof is included here only for the sake of completeness and for more advanced undergraduate students and first-year graduate students.

The proof makes use of **Euler's theorem.** According to Euler's theorem, if constant returns to scale prevail in production and if each factor is rewarded (paid) according to its productivity, the output produced is exhausted and just exhausted. Specifically, the marginal physical product of labor (MPL) times the amount of labor used in production (L) plus the marginal physical product of capital (MPK) times the amount of capital used in production (K) exactly equals the output produced. The same is true for commodity Y. In equation form, Euler's theorem in the production of commodity X can be expressed as

$$(MPL)(L) + (MPK)(K) = X \tag{5A-1}$$

Dividing both sides by L and rearranging:

$$X/L = MPL + (MPK)(K)/L \tag{5A-2}$$

Factoring out MPL:

$$X/L = MPL[(1 + K/L)(MPK/MPL)] \tag{5A-3}$$

With trade, Nation 1 produces at point B and Nation 2 produces at point B' in Figure 5.8. Since at points B and B', w/r is the same in both nations, MPK/MPL is also the same in both nations. We also know that at points B and B', K/L in the production of commodity X is the same in both nations. Finally, X/L is the average product of labor in the production of commodity X—and this is also the same in the two nations because of the assumptions of constant returns to scale and the same technology. As a result, the last remaining component (MPL) in Equation (5A-3) must also be the same in the production of commodity X *in both nations* if Equation (5A-3) is to hold.

Since the real wage is equal to MPL, the equality of MPL in the two nations means that real wages are the same in the two nations in the production of commodity X. With perfect

est (r) is also the same in both nations in the production of both commodities.

A5.4 Effect of Trade on the Short-Run Distribution of Income: The Specific-Factors Model

Suppose that in Nation 1 (the *L*-abundant nation) labor is mobile between industries but capital is not. Since labor is mobile, the wage of labor will be the same in the production of commodities X and Y in Nation 1. The equilibrium wage and the amount of labor employed in the production of X and Y in Nation 1 are given by the intersection of the value of the marginal product of labor curve in the production of X and Y. From microeconomic theory, we know that the value of the marginal product of labor in the production of X is equal to the price of commodity X times the marginal physical product of labor in the production of X. That is, $VMPL_X = (P_X)(MPL_X)$. Similarly, $VMPL_Y = (P_Y)(MPL_Y)$. We also know that if a firm employs more labor with a given amount of capital, *VMPL* declines because of the law of diminishing returns. Finally, to maximize profits, firms will employ labor until the wage they must pay equals the value of the marginal product of labor (i.e., until $w = VMPL$).

We can show the no-trade equilibrium wage and employment of labor in the production of commodities X and Y in Nation 1 with the aid of Figure 5.9. In the figure, the horizontal axis measures the total supply of labor available to Nation 1, and the vertical axis measures the wage rate. To begin with, concentrate on the $VMPL_X$ curve (which is read from left to right, as usual) and on the $VMPL_Y$ curve (which is read from right to left). The equilibrium wage rate is *ED* and is determined at the intersection of the $VMPL_X$ and $VMPL_Y$ curves. The wage rate is identical in the production of X and Y because of perfect labor mobility in the nation between the two industries. The amount *OD* of labor is used in the production of X, and the remainder, or *DO'*, is used in the production of Y.

Since Nation 1 (the *L*-abundant nation) has a comparative advantage in commodity X (the *L*-intensive commodity), the opening of trade increases P_X/P_Y. Since $VMPL_X = (P_X)(MPL_X)$, the increase in P_X shifts the $VMPL_X$ curve upward pro-portionately, by *EE'* to $VMPL'_X$. The wage rate increases less than proportionately, from *ED* to *E'D'*, and *DD'* units of labor shift from the production of Y to the production of X. Since w increases by less than the increase in P_X, w falls in terms of X but rises in terms of Y (since P_Y is unchanged). Thus, the effect of the increase in P_X on the real income of labor is ambiguous and depends on spending patterns. Workers who consume mainly commodity X will be worse off, while those who consume mainly commodity Y will be better off.

The rewards (r) to the specific factor (capital) change unambiguously, however. Since the specific capital in the production of commodity X has more labor to work with, $VMPK_X$ and r increase in terms of both commodities X and Y. On the other hand, since less labor is